Universidad Simón Bolívar
Departamento de Matemáticas
Puras y Aplicadas

Matemáticas II (MA1112)
Ene-Mar 2024
2^{do} Examen Parcial (35 %)

Duración: 1 hora 50 minutos

JUSTIFIQUE TODAS SUS RESPUESTAS

Pregunta 1. (5 ptos.) Halle $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{\left(5+2x+x^2\right)^3}}$

Pregunta 2. (3 ptos.) Halle $\displaystyle\int 3x\operatorname{sen}(x)\cos(x)\,\mathrm{d}x$

Pregunta 3. (2 ptos.) Halle $\operatorname{tgh}(x)$ sabiendo que $\operatorname{senh}(x) = 4/3$

Pregunta 4. (4 ptos.) Halle $\displaystyle\int \operatorname{sen}^2(3x)\cos^2(3x)\,\mathrm{d}x$

Pregunta 5. (4 ptos.) Halle $\displaystyle\int \operatorname{tg}^3(x)\sec^5(x)\,\mathrm{d}x$

Pregunta 6. (5 ptos.) Halle $\displaystyle\int 3x(\ln(x))^2\,\mathrm{d}x$

Pregunta 7. (5 ptos.) Halle $\displaystyle\int \frac{1+\operatorname{sen}(x)}{\operatorname{sen}(x)(1+\cos(x))}\,\mathrm{d}x$

Pregunta 8. (4 ptos.) Halle $\displaystyle\int \frac{2x^2+x+3}{(x+1)(x^2+1)}\,\mathrm{d}x$

Pregunta 9. (4 ptos.) Halle $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{25x^2-4}}$

SOLUCIONES

Pregunta 1. (5 ptos.) Halle $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{\left(5+2x+x^2\right)^3}}$

Completamos cuadrados para el polinomio dentro de la raíz cuadrada

$$5+2x+x^2 = x^2+2(x)(1)+1^2+4 = (x+1)^2+4$$

Así,

$$\int \frac{\mathrm{d}x}{\sqrt{\left(5+2x+x^2\right)^3}} = \int \frac{\mathrm{d}x}{\sqrt{\left((x+1)^2+4\right)^3}}$$

Aplicamos una sustitución trigonométrica:

$$x+1 = 2\tan(\Theta)\ ,\ \mathrm{d}x = 2\sec^2(\Theta)\,\mathrm{d}\Theta$$

Nos queda la integral:

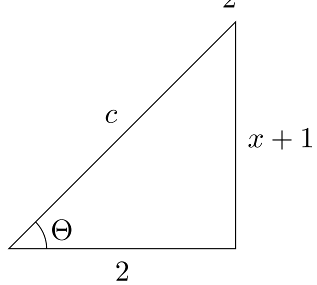
$$\int \frac{\mathrm{d}x}{\sqrt{\left((x+1)^2+4\right)^3}} = \int \frac{2\sec^2(\Theta)\,\mathrm{d}\Theta}{\sqrt{\left((2\tan(\Theta))^2+4\right)^3}} = \int \frac{2\sec^2(\Theta)}{\sqrt{\left(4\tan^2(\Theta)+4\right)^3}}\mathrm{d}\Theta = \int \frac{2\sec^2(\Theta)}{\sqrt{\left(4\tan^2(\Theta)+4\right)^3}}\mathrm{d}\Theta =$$

$$\int \frac{2\sec^2(\Theta)}{\sqrt{\left(4(\tan^2(\Theta)+1)\right)^3}}\mathrm{d}\Theta = \int \frac{2\sec^2(\Theta)}{\sqrt{\left(2^2\sec^2(\Theta)\right)^3}}\mathrm{d}\Theta = \int \frac{2\sec^2(\Theta)}{\sqrt{2^6\sec^6(\Theta)}}\mathrm{d}\Theta = \int \frac{2\sec^2(\Theta)}{2^3\sec^3(\Theta)}\mathrm{d}\Theta =$$

$$\int \frac{\mathrm{d}\Theta}{2^2\sec(\Theta)} = \frac{1}{4}\int \cos(\Theta)\,\mathrm{d}\Theta = \frac{1}{4}\,\mathrm{sen}(\Theta)+C$$

Devolvemos el cambio trigonométrico:

$$\tan(\Theta) = \frac{x+1}{2}$$



$\Rightarrow$

$$c^2 = (x+1)^2+2^2$$
$$c = \sqrt{5+2x+x^2}$$
$$\mathrm{sen}(\Theta) = \frac{x+1}{c} = \frac{x+1}{\sqrt{5+x+x^2}}$$

De aquí, $\displaystyle\frac{1}{4}\,\mathrm{sen}(\Theta)+C = \frac{x+1}{4\sqrt{5+x+x^2}}+C$

Finalmente, $\boxed{\displaystyle\int \frac{\mathrm{d}x}{\sqrt{\left(5+2x+x^2\right)^3}} = \frac{x+1}{4\sqrt{5+x+x^2}} + C}$

Pregunta 2. (3 ptos.) Halle $\displaystyle\int 3x\,\mathrm{sen}(x)\cos(x)\,\mathrm{d}x$

$$\int 3x\,\mathrm{sen}(x)\cos(x)\,\mathrm{d}x = \int \frac{3}{2}x\; 2\,\mathrm{sen}(x)\cos(x)\,\mathrm{d}x = \int \frac{3}{2}x\;\; \mathrm{sen}(2x)\,\mathrm{d}x = \int \frac{3}{4}x\; 2\,\mathrm{sen}(2x)\,\mathrm{d}x$$

Aplicamos integración por partes:

$$f(x) = \frac{3}{4}x \qquad \longrightarrow \qquad f'(x) = \frac{3}{4}$$

$$g'(x) = 2\,\mathrm{sen}(2x) \quad \longrightarrow \quad g(x) = -\cos(2x)$$

La integral nos queda:

$$\begin{aligned}\int \frac{3}{4}x\; 2\,\mathrm{sen}(2x)\,\mathrm{d}x &= -\frac{3}{4}x\cos(2x) + \int \frac{3}{4}\cos(2x)\,\mathrm{d}x \\ &= -\frac{3}{4}x\cos(2x) + \frac{3}{8}\int 2\cos(2x)\,\mathrm{d}x \\ &= -\frac{3}{4}x\cos(2x) + \frac{3}{8}\,\mathrm{sen}(2x) + C\end{aligned}$$

Finalmente, $\boxed{\displaystyle\int 3x\,\mathrm{sen}(x)\cos(x)\,\mathrm{d}x = \frac{3}{8}\,\mathrm{sen}(2x) - \frac{3}{4}x\cos(2x) + C}$

Pregunta 3. (2 ptos.) Hallte $\mathrm{tgh}(x)$ sabiendo que $\mathrm{senh}(x) = 4/3$

Es sabido que $\mathrm{tgh}(x) = \dfrac{\mathrm{senh}(x)}{\cosh(x)}$

A su vez, $\cosh(x) = \pm\sqrt{1+\mathrm{senh}^2(x)} \underset{\substack{\uparrow \\ \boxed{\cosh(x)\geq 1\ \forall\ x\in\mathbb{R}}}}{=} \sqrt{1+\mathrm{senh}^2(x)}$

Por lo tanto, $\mathrm{tgh}(x) = \dfrac{\mathrm{senh}(x)}{\cosh(x)} = \dfrac{\mathrm{senh}(x)}{\sqrt{1+\mathrm{senh}^2(x)}} = \dfrac{4/3}{\sqrt{1+(4/3)^2}} = \dfrac{4/\cancel{3}}{5/\cancel{3}} = \dfrac{4}{5}$

Finalmente, $\boxed{\mathrm{tgh}(x) = \dfrac{4}{5}}$

Pregunta 4. (4 ptos.) Halle $\int \operatorname{sen}^2(3x)\cos^2(3x)\,\mathrm{d}x$

$$\int \operatorname{sen}^2(3x)\cos^2(3x)\,\mathrm{d}x = \int \left(\frac{1-\cos(6x)}{2}\right)\left(\frac{1+\cos(6x)}{2}\right)\mathrm{d}x = \frac{1}{4}\int \big(1-\cos^2(6x)\big)\,\mathrm{d}x =$$

$$= \frac{1}{4}\int \left(1 - \frac{1+\cos(12x)}{2}\right)\mathrm{d}x = \frac{1}{4}\int \left(\frac{1-\cos(12x)}{2}\right)\mathrm{d}x = \frac{1}{8}\int \mathrm{d}x - \frac{1}{96}\int 12\cos(12x)\,\mathrm{d}x =$$

$$= \frac{1}{8}x - \frac{1}{96}\operatorname{sen}(12x) + C$$

Finalmente, $\boxed{\int \operatorname{sen}^2(3x)\cos^2(3x)\,\mathrm{d}x = \frac{1}{8}x - \frac{1}{96}\operatorname{sen}(12x) + C}$

Pregunta 5. (4 ptos.) Halle $\int \operatorname{tg}^3(x)\sec^5(x)\,\mathrm{d}x$

$$\int \operatorname{tg}^3(x)\sec^5(x)\,\mathrm{d}x = \int \operatorname{tg}^2(x)\sec^4(x)\operatorname{tg}(x)\sec(x)\,\mathrm{d}x = \int \big(\sec^2(x)-1\big)\sec^4(x)\operatorname{tg}(x)\sec(x)\,\mathrm{d}x$$

Cambio de variable:

$$u = \sec(x)\ ,\ \mathrm{d}u = \sec(x)\operatorname{tg}(x)\,\mathrm{d}x$$

Nos queda la integral:

$$\int \big(\sec^2(x)-1\big)\sec^4(x)\operatorname{tg}(x)\sec(x)\,\mathrm{d}x = \int (u^2-1)u^4\,\mathrm{d}u = \int (u^6-u^4)\,\mathrm{d}u = \frac{1}{7}u^7 - \frac{1}{5}u^5 + C =$$

$$= \frac{1}{7}\sec^7(x) - \frac{1}{5}\sec^5(x) + C$$

Finalmente, $\boxed{\int \operatorname{tg}^3(x)\sec^5(x)\,\mathrm{d}x = \frac{1}{7}\sec^7(x) - \frac{1}{5}\sec^5(x) + C}$

Pregunta 6. (4 ptos.) Halle $\int 3x\big(\ln(x)\big)^2\,\mathrm{d}x$

Aplicamos integración por partes:

$$f(x) = \big(\ln(x)\big)^2 \quad\longrightarrow\quad f'(x) = \frac{2\ln(x)}{x}$$

$$g'(x) = 3x \qquad\longrightarrow\qquad g(x) = \frac{3}{2}x^2$$

La integral nos queda:

$$\int 3x\big(\ln(x)\big)^2\,\mathrm{d}x = \frac{3}{2}x^2\big(\ln(x)\big)^2 - \underbrace{\int 3x\ln(x)\,\mathrm{d}x}_{\mathrm{I}}$$

Aplicamos integración por partes para I:

$$f(x) = \ln(x) \quad \longrightarrow \quad f'(x) = \frac{1}{x}$$

$$g'(x) = 3x \quad \longrightarrow \quad g(x) = \frac{3}{2}x^2$$

La integral nos queda:

$$\mathrm{I} = \frac{3}{2}x^2\ln(x) - \frac{3}{2}\int x\,\mathrm{d}x = \frac{3}{2}x^2\ln(x) - \frac{3}{4}x^2 + C_1$$

Volviendo a la integral original:

$$\int 3x\big(\ln(x)\big)^2\,\mathrm{d}x = \frac{3}{2}x^2\big(\ln(x)\big)^2 - \mathrm{I} = \frac{3}{2}x^2\big(\ln(x)\big)^2 - \frac{3}{2}x^2\ln(x) + \frac{3}{4}x^2 + C$$

Finalmente, $\boxed{\int 3x\big(\ln(x)\big)^2\,\mathrm{d}x = \frac{3}{2}x^2\left(\big(\ln(x)\big)^2 - \ln(x) + \frac{1}{2}\right) + C}$

Pregunta 7. (5 ptos.) Halle $\displaystyle\int \frac{1+\operatorname{sen}(x)}{\operatorname{sen}(x)\big(1+\cos(x)\big)}\,\mathrm{d}x$

Aplicamos cambio universal:

$$\tan\left(\frac{x}{2}\right) = u\ ,\ \operatorname{sen}(x) = \frac{2u}{1+u^2}\ ,\ \cos(x) = \frac{1-u^2}{1+u^2}\ ,\ \mathrm{d}x = \frac{2}{1+u^2}\,\mathrm{d}u$$

Nos queda la integral:

$$\int \frac{1+\operatorname{sen}(x)}{\operatorname{sen}(x)\big(1+\cos(x)\big)}\,\mathrm{d}x = \int \frac{1+\frac{2u}{1+u^2}}{\frac{\cancel{2}u}{\cancel{1+u^2}}\left(1+\frac{1-u^2}{1+u^2}\right)}\frac{\cancel{2}}{\cancel{1+u^2}}\,\mathrm{d}u = \int \frac{\frac{1+u^2+2u}{\cancel{1+u^2}}}{u\cdot\frac{1+\cancel{u^2}+1-\cancel{u^2}}{\cancel{1+u^2}}}\,\mathrm{d}u = \int \frac{u^2+2u+1}{2u}\,\mathrm{d}u =$$

$$= \int\left(\frac{u}{2}+1+\frac{1}{2u}\right)\mathrm{d}u = \frac{1}{4}u^2 + u + \frac{1}{2}\ln|u| + C = \frac{1}{4}\tan^2\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2}\right) + \frac{1}{2}\ln\left|\tan\left(\frac{x}{2}\right)\right| + C$$

Finalmente, $\boxed{\int \frac{1+\operatorname{sen}(x)}{\operatorname{sen}(x)\big(1+\cos(x)\big)}\,\mathrm{d}x = \frac{1}{4}\tan^2\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2}\right) + \frac{1}{2}\ln\left|\tan\left(\frac{x}{2}\right)\right| + C}$

Pregunta 8. (4 ptos.) Halle $\displaystyle\int \frac{2x^2+x+3}{(x+1)(x^2+1)}\,\mathrm{d}x$

Como el grado del polinomio del numerador es menor que el del denominador, aplicamos el método de fracciones parciales:

$$\frac{2x^2+x+3}{(x+1)(x^2+1)} = \frac{\mathrm{A}}{x+1} + \frac{\mathrm{B}x+\mathrm{C}}{x^2+1} = \frac{\mathrm{A}(x^2+1)+(\mathrm{B}x+\mathrm{C})(x+1)}{(x+1)(x^2+1)}$$

$$\begin{aligned}
\rightarrow\ 2x^2+x+3 &= \mathrm{A}x^2+\mathrm{A}+\mathrm{B}x^2+\mathrm{B}x+\mathrm{C}x+\mathrm{C}\\
&= (A+B)x^2+(B+C)x+(A+C)\\
&\Rightarrow \begin{cases}\mathrm{A}+\mathrm{B}=2\\ \mathrm{B}+\mathrm{C}=1\\ \mathrm{A}+\mathrm{C}=3\end{cases} \quad\Rightarrow\ \mathrm{A}=2,\ \mathrm{B}=0,\ \mathrm{C}=1
\end{aligned}$$

Por lo tanto, nos queda la integral:

$$\int \frac{2x^2+x+3}{(x+1)(x^2+1)}\,\mathrm{d}x = \int \frac{2}{x+1}\,\mathrm{d}x + \int \frac{1}{x^2+1}\,\mathrm{d}x = 2\ln|x+1| + \arctan(x) + C$$

Finalmente, $\boxed{\displaystyle\int \frac{2x^2+x+3}{(x+1)(x^2+1)}\,\mathrm{d}x = 2\ln|x+1| + \arctan(x) + C}$

Pregunta 9. (4 ptos.) Halle $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{25x^2-4}}$

$$\int \frac{\mathrm{d}x}{\sqrt{25x^2-4}} = \int \frac{\mathrm{d}x}{\sqrt{(5x)^2-4}}$$

Aplicamos sustitución trigonométrica:

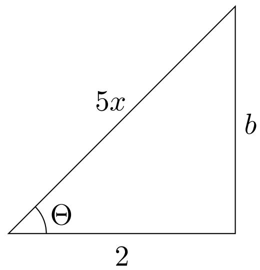
$$5x = 2\sec(\Theta)\ ,\ \mathrm{d}x = \frac{2}{5}\sec(\Theta)\tan(\Theta)\,\mathrm{d}\Theta$$

Nos queda la integral:

$$\int \frac{\mathrm{d}x}{\sqrt{(5x)^2-4}} = \int \frac{{}^2\!/\!{}_5\sec(\Theta)\tan(\Theta)}{\sqrt{(2\sec(\Theta))^2-4}}\,\mathrm{d}\Theta = \frac{\cancel{2}}{5}\int \frac{\sec(\Theta)\tan(\Theta)}{\cancel{2}\sqrt{\sec^2(\Theta)-1}}\,\mathrm{d}\Theta = \frac{1}{5}\int \frac{\sec(\Theta)\tan(\Theta)}{\sqrt{\tan^2(\Theta)}}\,\mathrm{d}\Theta =$$
$$= \frac{1}{5}\int \frac{\sec(\Theta)\cancel{\tan(\Theta)}}{\cancel{\tan(\Theta)}}\,\mathrm{d}\Theta = \frac{1}{5}\int \sec(\Theta)\,\mathrm{d}\Theta = \frac{1}{5}\ln|\sec(\Theta)+\tan(\Theta)| + C_1$$

Devolviendo el cambio trigonométrico:

$$\sec(\Theta) = \frac{5x}{2}$$



$$\Rightarrow$$

$$b^2 = (5x)^2 - 2^2$$
$$b = \sqrt{25x^2 - 4}$$
$$\tan(\Theta) = \frac{\sqrt{25x^2-4}}{2}$$

De aquí,

$$\frac{1}{5}\ln|\sec(\Theta) + \tan(\Theta)| + C_1 = \frac{1}{5}\ln\left|\frac{5x}{2} + \frac{\sqrt{25x^2-4}}{2}\right| + C_1 = \frac{1}{5}\ln\left|\frac{5x+\sqrt{25x^2-4}}{2}\right| + C_1 =$$
$$= \frac{1}{5}\ln|5x+\sqrt{25x^2-4}| \underbrace{-\frac{1}{5}\ln(2) + C_1}_{\text{Constante } C} = \frac{1}{5}\ln|5x+\sqrt{25x^2-4}| + C$$

Finalmente, $\boxed{\displaystyle\int \frac{\mathrm{d}x}{\sqrt{25x^2-4}} = \frac{1}{5}\ln|5x+\sqrt{25x^2-4}| + C}$

Este parcial fue digitalizado en LaTeX por **Daniel Quijada** para **GECOUSB**

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Cualquier error en la resolución de los ejercicios, notificar a **20-10518@usb.ve**